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# Interfacial contribution to thickness dependent in-plane anisotropic magnetoresistance

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We have studied in-plane anisotropic magnetoresistance (AMR) in cobalt films with overlayers having designed electrically interface transparency. With an electrically opaque cobalt/overlayer interface, the AMR ratio is shown to vary in inverse proportion to the cobalt film thickness; an indication that in-plane AMR is a consequence of anisotropic scattering with both volume and interfacial contributions. The interface scattering anisotropy opposes the volume scattering contribution, causing the AMR ratio to diminish as the cobalt film thickness is reduced. An intrinsic interface effect explains the significantly reduced AMR ratio in ultra-thin films. © 2015 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License. [<http://dx.doi.org/10.1063/1.4937556>]

## I. INTRODUCTION

Anisotropic magnetoresistance (AMR) was first observed by William Thomson<sup>1</sup> and, despite concerted study over almost 150 years<sup>2</sup>, detailed understanding of the physical phenomenology underpinning AMR continues to emerge.<sup>3–5</sup> In ferromagnetic (FM) metals AMR is considered to be a consequence of the spin-orbit interaction (SOI) within the *bulk* and is manifest as a dependence of the electrical resistivity,  $\rho$ , on the angle  $\theta$  between current density  $J$  and magnetization  $M$ . In polycrystalline FM transition-metals it has been shown<sup>2,6</sup> that a simple variation of  $\rho$  with  $\theta$  goes as  $\rho(\theta) = \rho_{\perp} + \Delta\rho\cos^2\theta$ , where  $\rho_{\perp}$  is the transverse ( $\theta = \pi/2$ ) resistivity, and  $\Delta\rho$  is the difference between longitudinal ( $\rho_{\parallel} = \rho(\theta = 0)$ ) and transverse resistivities,  $\Delta\rho = \rho_{\parallel} - \rho_{\perp}$ . Here the AMR ratio is defined as;  $\text{AMR} = \Delta\rho/\rho_{\perp}$ , as both  $\rho_{\parallel}$  and  $\rho_{\perp}$  correspond to well-defined saturated magnetization states.<sup>7</sup>

Interest in anisotropic magnetotransport effects has had a resurgence recently due to the discoveries of novel phenomena such as the tunnelling AMR (TAMR) in FM semiconductors,<sup>8</sup> and metals<sup>9</sup> Coulomb-blockade AMR,<sup>10</sup> TAMR in antiferromagnets,<sup>11</sup> spin-Hall magnetoresistance (SHMR),<sup>12</sup> and ‘hybrid’ magnetoresistance.<sup>13</sup> In addition to these exotic anisotropic effects, the conventional AMR in FM metals still continues to present unexplained behaviour.

The standard description for the characteristic reduction in AMR ratio with decreasing FM thickness, that it is due simply to the increase in resistivity in the Fuchs-Sondheimer (F-S) regime<sup>14</sup> with  $\Delta\rho$  being independent of FM film thickness,<sup>2,13</sup> does not scale to ultra-thin films; for thicknesses below  $\sim 10$  nm,  $\Delta\rho$  actually depends strongly on FM film thickness.<sup>5,15</sup>

Recently, an interfacial contribution to the out-of-plane AMR, dubbed anisotropic interfacial magnetoresistance, which manifests as a small thickness-dependent difference between the AMR for magnetization rotated *in-plane* and *out-of-plane* has been found in Pt/Co/Pt trilayers<sup>4</sup> and Co/Pd

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multilayers.<sup>16</sup> This anisotropic interface magnetoresistance has since been suggested to be explained in terms of a ‘hybrid’ magnetoresistance as the result of interfacial proximity polarisation of Pt or Pd adjacent to a FM.<sup>13</sup>

There is a significant interest in effects originating in interfacial SOI. The Rashba-torque in layered thin-film FM systems with structural inversion asymmetry<sup>17</sup> may be thought of as the converse of the AMR. AMR in Rashba systems has been studied theoretically, and it has been shown that the Rashba SOI produces *in-plane* AMR in the FM layer with the *same sign*, i.e.  $\rho_{\parallel} > \rho_{\perp}$ , and *symmetry*, i.e.,  $\cos^2 \theta$  dependence, as ‘conventional’ AMR.<sup>18</sup> Unconventional AMR in an adjacent layer with strong SOI may also arise due to Rashba SOI, but has a different angular symmetry to conventional AMR.<sup>19</sup> Enhanced AMR due to *structurally symmetric* adjacent layers with strong SOI has also been suggested by Liu *et al.*<sup>3</sup> Overall, such results suggest that the *in-plane* AMR in a single ultra-thin FM metal film may have a previously unconsidered *interfacial* contribution, as a result of anisotropic interface scattering and/or interfacial spin-orbit interactions. Here we demonstrate the interfacial contribution to the *in-plane* AMR in a FM layer.

We study the FM film-thickness dependence of the AMR in cobalt films deposited onto seed-layers that promote strong fcc(111) [hcp(0001)] texture in the cobalt for all thicknesses.<sup>20</sup> This unambiguously isolates any interfacial contribution to AMR from effects due to variations in the film microstructure and/or texture between pairs of samples with different overlayers. Microstructural variation in the cobalt layers would occur were we to use Ir/Co/Ir multilayers; due to the lattice mismatch between Ir and Co, Co grown on Ir will have a very different density of crystalline defects. To isolate the contribution to the AMR from a single designed interface, structures with overlayers of either copper or iridium were used. Due to the similarity between the (majority spin) electronic band-structure of cobalt and copper<sup>21</sup> the structures with copper overlayers have electrically transparent interfaces, and with the copper seed layers preserve structural inversion symmetry. The different electronic band-structure of iridium compared to cobalt means that structures with iridium overlayers have more electrically opaque interfaces and broken structural inversion symmetry. Iridium also has strong SOI (larger atomic number), which should enhance any Rashba contribution to the AMR. These structures allow the isolation of the *in-plane* AMR contribution due to scattering at an electrically opaque interface — using the Co/Ir interface as an illustrative model system.

## II. EXPERIMENTAL

The multilayer structures were deposited onto Si substrates, with  $\sim 100$  nm thermal oxide coating, by a combination of dc and rf magnetron sputtering. The structures consist of Si/SiO/Ta[3]/Cu[3]/Co[ $t_{\text{Co}}$ ]/overlayer[3]/Ta[3] (layer thicknesses in nm units) with  $t_{\text{Co}}$  ranging from 2 nm to 55 nm: These are referred to as Co([ $t_{\text{Co}}$ ])/overlayer for brevity. Layer thicknesses were confirmed using x-ray reflectivity. Current flow through Cu/Co/Ir(Cu) layers only is assumed as the (amorphous) Ta seed and (atmospherically oxidised) capping layers are highly resistive. Resistivity measurements were made using a dc in-line 4-probe method using samples diced into  $15 \text{ mm} \times 2 \text{ mm}$  pieces. Resistivity was calculated from measured resistance  $R = V/I$  as  $\rho = c \times RI/wt$  where  $w$  is the sample width,  $l$  the voltage probe spacing ( $\sim 7$  mm) and  $t$  the total sample thickness. Due to the sample and measurement geometry the geometrical correction factor  $c$  may be taken as unity. Magnetic characterization (not shown) was performed using magneto-optical Kerr effect and SQUID magnetometry. All measurements were made at room temperature. Samples showed in-plane magnetization for the entire range of thickness studied, thus we expect not contribution to the magnetoresistance from anomalous Hall effect. X-ray diffraction and ferromagnetic resonance spectroscopy measurements for these samples are described in Reference 20. A schematic of the sample and electrical measurement geometry is shown inset to Fig. 1(c).

## III. RESULTS AND DISCUSSION

Typical AMR behavior was observed, the  $\cos^2 \theta$  dependence (not shown) indicating that any Rashba contribution to *in-plane* AMR with Ir(Cu) overlayers is negligible.<sup>19</sup> Inset to Fig. 1(a) are

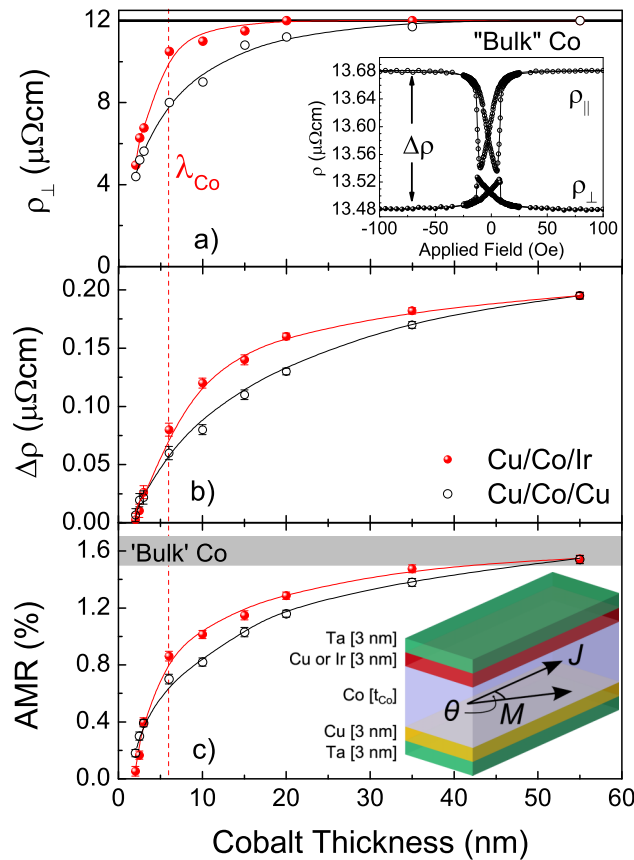


FIG. 1. Dependence of the (a) transverse resistivity,  $\rho_{\perp}$ , (b)  $\Delta\rho = \rho_{\parallel} - \rho_{\perp}$ , and (c) AMR ratio,  $\Delta\rho/\rho_{\perp}$ , on Co film thickness in structures with Cu and Ir overlayers. Inset to (a) is a representative resistivity-field response in longitudinal ( $\rho_{\parallel}$ ,  $\theta = 0$ ) and transverse ( $\rho_{\perp}$ ,  $\theta = \pi/2$ ) orientations, and inset to (c) is a schematic of the sample structure and measurement geometry. Typical values for  $\rho$ ,  $\lambda_{\text{Co}}$ , and AMR ratio for 'bulk' cobalt are indicated, error-bars primarily represent uncertainty in the measurement of the sample dimensions and are smaller than the data markers in (a), and solid lines through the data are a guide to the eye.

representative longitudinal and transverse magnetoresistance for Co[35]/Cu. The coercivity,  $H_C$ , and the resistivity at  $H_C$  depend upon the relative orientation of current and field; indicating a weak magnetic anisotropy and resulting in an angular dependence of  $H_C$ . The demagnetized state is not a single, well-defined, magnetic state.

Fig. 1(a) shows the thickness dependence of the transverse resistivity,  $\rho_{\perp}$ , for both the Co/Cu and Co/Ir series. In each case the resistivity for large thickness corresponds to the bulk room-temperature resistivity of Co,  $\sim 12 \mu\Omega \text{ cm}$ , typical of sputtered polycrystalline material. The bulk mean-free-path for Co was estimated from this as  $\lambda_{\text{Co}} \approx 6 \text{ nm}$ , also indicated in Fig. 1. At the smallest  $t_{\text{Co}}$  the resistivity tends toward that of Cu,  $\sim 2 \mu\Omega \text{ cm}$ , indicating increased current-shunting through the Cu seed-layer (and overlayer in Co/Cu). This is more pronounced in the Co/Cu series. Note that a simple parallel-resistor model for individual layers in the multilayer structure cannot be used here to describe either the cobalt thickness dependence of  $\rho$  or  $\Delta\rho$ . This dependence of resistivity on  $t_{\text{Co}}$  also makes a simple F-S analysis inappropriate and it offers no information on the interface scattering.<sup>22</sup> However, since  $\rho_{\perp}$  for the Co/Ir series is close to the 'bulk' Co resistivity until  $t_{\text{Co}} \sim 6 \text{ nm}$  ( $\sim \lambda_{\text{Co}}$ ) while the resistivity falls below  $t_{\text{Co}} \sim 30 \text{ nm}$  for Co/Cu it suggests such a variation in  $t_{\text{Co}}$  dependence may indicate a higher degree of 'specular' reflection at the Co/Ir interface compared to the Co/Cu interface.<sup>23</sup> Studies on giant magnetoresistance (GMR) have shown the Co/Cu interface has a high transparency for majority  $s$ -like carriers due to the similarity in the electronic structure across the interface.<sup>21</sup> These states dominate the conduction in Co. In contrast, the electronic states across a Co/Ir interface are dissimilar and the Co/Ir interface is far less transparent, resulting in orders of magnitude smaller GMR.<sup>24</sup> This

supports the idea that scattering at the opaque Co/Ir interface dominates over that at the transparent Co/Cu interface.

Figure 1(b) shows  $\Delta\rho$  for the same series of films. For both Co/Cu and Co/Ir,  $\Delta\rho$  falls with decreasing  $t_{\text{Co}}$ . However,  $\Delta\rho$  remains larger for the Co/Ir series than the Co/Cu, with the exception of the smallest  $t_{\text{Co}}$ . A similar dependence is shown in Fig. 1(c) for the AMR ratio. It is important to note that divergence from the bulk values start at the same  $t_{\text{Co}}$  for AMR and  $\Delta\rho$  but this divergence is seen at different  $t_{\text{Co}}$  for  $\rho_{\perp}$ ; suggesting that the thickness dependence of the AMR is more strongly related to that of  $\Delta\rho$  than  $\rho_{\perp}$ . The thickness dependence of AMR is conventionally suggested to be dominated by the F-S thickness dependence of the resistivity, with  $\Delta\rho$  being independent of thickness.<sup>2,13</sup> Boltzmann transport analysis with non-specular interface scattering, where the AMR is included via an anisotropic *bulk* mean-free-path, suggests that  $\Delta\rho$  *diverges* with decreasing thickness; but, this divergence is slower than that of  $\rho$ , resulting in the observed reduction in AMR at low thickness.<sup>23</sup> Neither of these suggested explanations for the thickness dependence of AMR can apply in the present case where  $\rho$  and  $\Delta\rho$  cannot diverge, due to current shunting. However, AMR still reduces with decreasing film thickness in a similar fashion to other observations. A new description is required in order to explain the thickness dependence of the AMR. This data shows that the AMR ratio in the ultra-thin film limit is determined predominantly by the thickness-dependence of  $\Delta\rho$  rather than  $\rho_{\perp}$  (or  $\rho_0$ ). The AMR ratio for films with the largest  $t_{\text{Co}}$  corresponds to the ‘bulk’ Co AMR,<sup>4</sup> and indicated by the shaded region in Fig. 1(c). The decrease in both  $\Delta\rho$  and the AMR ratio with decreasing FM film thickness is seen in single films,<sup>5,15</sup> and multilayers<sup>3</sup> and is, in general, not entirely due to current shunting through under/overlayers at smaller FM thicknesses,<sup>16</sup> contrary to the suggestion in Ref. 4. Were current shunting to be the dominant effect, we would expect that both  $\Delta\rho$  and  $\rho$  would be modified from their bulk values at the same cobalt layer thickness. The crossover at low thickness can not be explained due to magnon scattering.<sup>5</sup>

Liu *et al.*<sup>3</sup> studied AMR in multilayers of Ta/(Pt)/NiFe/(Pt)/Ta and observed an enhancement similar to that shown here for Co/Ir in Fig. 1(c), and attributed to interfacial SOI. However, they did not study films below 5 nm. An advantage here is that the insertion of Ir only above the FM layer causes no microstructural modification, unlike the insertion of a Pt layer below NiFe.<sup>3</sup>

An alternative explanation is that Pt (or Ir) layers prevent the formation of magnetically dead layers at the FM/Ta interfaces. Such dead-layers should affect the AMR by reducing  $\Delta\rho$ , via the reduced FM volume, whilst not significantly changing  $\rho_{\perp}$  (or  $\rho_0$ ). Although this can account for the enhanced AMR ratio in Ta/(Pt)/NiFe/(Pt)/Ta structures with Pt layers inserted,<sup>3</sup> no such dead-layers are expected to form at the Co/Cu (or Co/Ir) interfaces here. This was confirmed by SQUID magnetometry (not shown), and is well-known from extensive studies of GMR and interlayer exchange coupling in these material systems.<sup>21,24</sup> Also, thin Pt layers are strongly magnetically polarized by proximity with an adjacent FM, which may enhance the effective FM volume.<sup>25</sup> and leading to hybrid proximity magnetoresistance.<sup>13</sup> Unlike Pt, both Cu and Ir exhibit at most only a weak oscillatory polarization, so cannot significantly modify the total FM volume.<sup>21,24</sup> The absence of measurable proximity polarization of adjacent Cu and Ir layers was confirmed by SQUID and ferromagnetic resonance characterization.<sup>20</sup> Indeed, ensuring that no interfacial magnetic dead layers or proximity polarization form was crucial here and neither interfacial magnetic dead-layers nor proximity polarization account for the enhanced AMR in Co/Ir over that in Co/Cu observed here.

To understand the interfacial contribution to the AMR Fig. 2 shows  $\Delta\rho$  and the AMR ratio as a function of *inverse* cobalt thickness for both sets of films. Such plots are commonly used to show the interfacial contribution to phenomena such as magnetic anisotropy and magnetostriction<sup>26–28</sup> which also arise from SOI<sup>29</sup>. For example, the effective magnetocrystalline anisotropy  $K_{\text{eff}}$  in thin FM films may be decomposed into volume and interfacial terms,  $K_{\text{eff}} = K_{\text{vol}} + K_{\text{int}}/t_{\text{FM}}$ , where  $t_{\text{FM}}$  is the FM film thickness. The interfacial term,  $K_{\text{int}}$ , is found to have *opposite sign* to the volume term,  $K_{\text{vol}}$  and cancellation occurs, resulting in near-zero effective magnetocrystalline anisotropy at a characteristic FM film thickness of  $\sim 1\text{--}1.3$  nm.<sup>26,30</sup>

For the Co/Cu series of films, Fig. 2 shows a strongly non-linear dependence of  $\Delta\rho$  and AMR on  $1/t_{\text{Co}}$ , suggesting that scattering at the Co/Cu interfaces does not play a dominant role in determining the thickness dependence. Current shunting dominates because of the transparency of the interfaces to the majority *s*-like states which primarily contribute to conduction. Co/Ir films have a lower interface



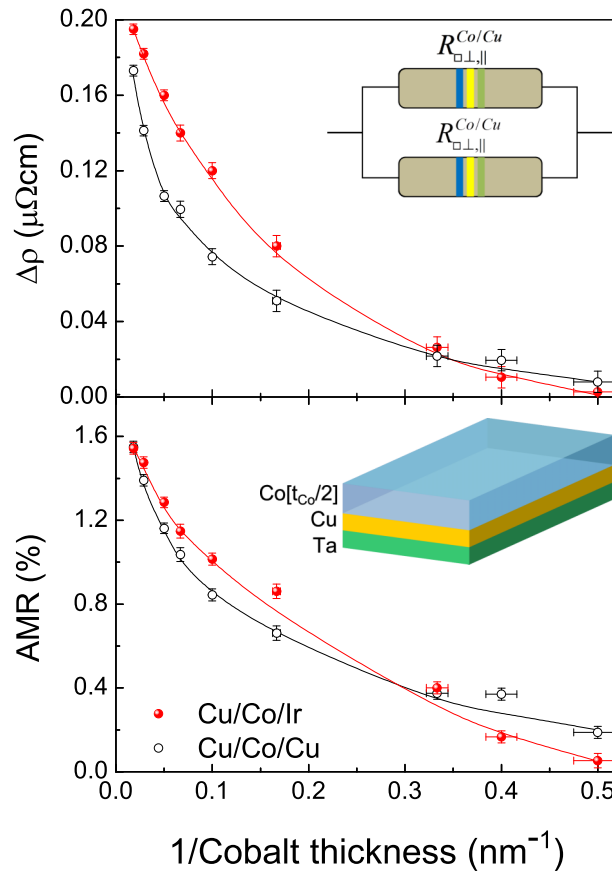


FIG. 2. Inverse cobalt thickness dependence of (upper)  $\Delta\rho$  and (lower) AMR for Co/Cu and Co/Ir structures. Solid lines through the data are a guide to the eye. Insets show a schematic Co/Cu 'half-structure', and the parallel resistor model employed.

transparency for conducting states at the Co/Ir interface, and current shunting should be reduced. A more linear dependence of both  $\Delta\rho$  and the AMR on  $1/t_{\text{Co}}$  was found, suggesting an interfacial contribution to the anisotropic scattering that gives rise to the AMR.

Difficulty arises in experimentally determining the exact contribution of a single interface. For studies of magnetic anisotropy, systems can be designed with one dominant interface.<sup>31,32</sup> The Cu/Co/Ir provides an analogous system with a single dominant interfacial contribution to AMR. To demonstrate this any contribution due to scattering at the Co/Cu interface or the current-shunting effect of the Cu seed-layer must be accounted for.

This was done by effectively dividing the symmetric Co/Cu structure in to two parts of equal cobalt thickness. Then only one of these parts, with a single *transparent* Co/Cu interface was considered as shown schematically inset in Fig. 2. The data in Fig. 2 was analysed using this approach to obtain  $\rho_{\perp}$  and  $\rho_{\parallel}$  for these Co/Cu 'half-structures' using a simple parallel resistor model for the sheet resistances,  $R_{\square\perp}$  and  $R_{\square\parallel}$ . This is equivalent to setting the imagined cobalt/vacuum interface as specularly reflective within the F-S type model.<sup>23</sup>

Making a similar division of the asymmetric Co/Ir structures, these then consist of one of the Co/Cu 'half-structures' and a Co/Ir 'half-structure'. Using the parallel resistor model and removing the contribution of the Co/Cu half-structure the AMR for half-structures containing a single Co/Ir interface was obtained. Note that this approach does not consider differences in current distribution in the Co layer between Co/Cu and Co/Ir structures, which will differ increasingly as the Co film thickness decreases. More complex calculations within a F-S type model for such trilayers<sup>4</sup> do not yield more beneficial or physically meaningful results, since the F-S model for a single layer explicitly

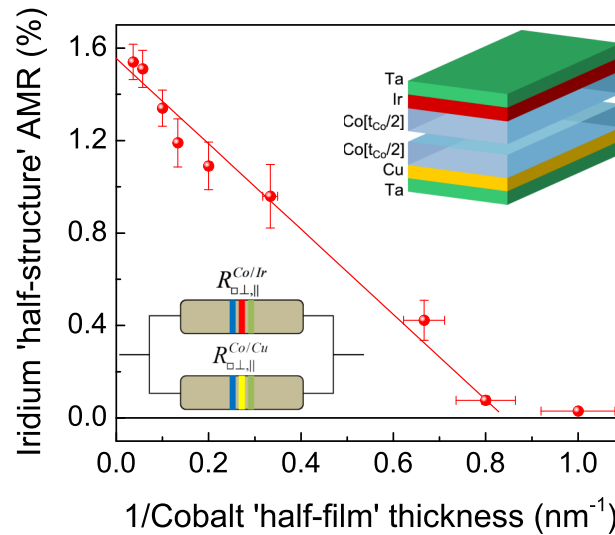


FIG. 3. Inverse cobalt thickness dependence of AMR for a Co/Ir 'half-structure' extracted from the parallel resistor model as shown inset. The solid line is a fit excluding the sample with thinnest Co layer. The 'characteristic' thickness for which volume and interfacial scattering terms may cancel falls within the region  $t_{\text{Co}}/2 = (1.2 \pm 0.02)$  nm.

considering only *bulk* scattering anisotropy predicts the opposite behavior of  $\Delta\rho$  to that found experimentally.<sup>23</sup>

The inverse cobalt (half-)film thickness dependence of the AMR for Co/Ir 'half-structures' is shown in Fig. 3. A linear dependence upon inverse (half-)film thickness was found, indicating an interfacial contribution to anisotropic scattering. It suggests that here the scattering anisotropy for the interface is opposite to that of the volume contribution. For the lowest  $t_{\text{Co}}$  measured our simple model breaks down as current-shunting through the Cu seed and overlayers makes the assumption of consistent current distribution invalid. The linear fit to the data, excluding the sample with  $t_{\text{Co}} = 2$  nm, defines an approximate cobalt (half-)film thickness,  $t_{\text{Co}}/2 = (1.2 \pm 0.02)$  nm, where the Co/Ir interfacial contribution to anisotropic scattering may be expected to cancel the Co volume contribution.

This simple analysis provides an explanation for the interfacial contribution to the thickness dependence of the in-plane AMR: the in-plane AMR in ultra-thin film multilayers is a result not only of *bulk* anisotropic scattering as considered previously,<sup>23</sup> but also contains anisotropic interface scattering contributions. This is consistent with, but distinct from, the finding by Kobs *et al.*,<sup>4</sup> who suggest a contribution of interface scattering to out-of-plane AMR. As the film thickness decreases the anisotropic interface scattering term becomes increasingly important until, at some critical thickness,  $\Delta\rho$  and the AMR ratio vanish. We suggest that such interface scattering may prevent the divergence of  $\Delta\rho$  at low thickness which is predicted by F-S type theory.<sup>23</sup>

As a result of anisotropic interface scattering, the AMR may vanish at a critical FM film thickness,  $\sim 2.4$  nm for the Co/Ir material system. The effective polarisation in NiFe has also recently been found to extrapolate to zero at a film thickness  $\sim 2.3$  nm.<sup>33</sup> Maintaining a high AMR ratio at low film thickness requires opaque interfaces to prevent current-shunting and a materials combination which suppresses dead-layer formation. This informs on the design of AMR sensors. The interfacial scattering contribution to the AMR presents a fundamental physical limit to scalability as, even under these conditions, the AMR diminishes as film thickness decreases.

#### IV. SUMMARY

In summary, it has been shown, using structures containing an electrically opaque – in this case Co/Ir – interface, that there is an interfacial contribution to the anisotropic scattering that affects the in-plane AMR. This interfacial contribution can act to cancel the bulk scattering anisotropy.

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